Top-pair production at Hadron Colliders:

newest developments

Alexander Mitov

Yang Institute for Theoretical Physics
SUNY Stony Brook

Work with Michael Czakon
arXiv:0812.0353
arXiv:0811.4119
and in progress…

Current status

The state of the art is NLO QCD corrections.

Original results derived long ago (20 years):

Nason, Dawson, Ellis (1988-90)
Beenakker, Kuijf, van Neerven, Smith (1989)
Beenakker, van Neerven, Meng, Schuler, Smith (91)
Mangano, Nason, Ridolfi (1992)

New results (2 months ago):

M. Czakon, A.M. (2008)

- Various observables:

  a) Differential:
  single particle inclusive,
  pair-invariant mass distribution,
  etc.

  b) Fully inclusive (until two months ago – numerical; now analytic)

- Relevance of the differential vs the total cross section:

<u>For not too strong cuts, the NLO effect is on normalization, not shapes !</u>

Current status

Second source: NLL soft gluon (threshold) resummation.

The only source of new information in top production in the last 10 years

- Various observables:

  a) Differential:
  single particle inclusive,
  pair-invariant mass distribution,
  etc.

  Developed: Sterman et al mid-90's
  Applied: Kidonakis, Laenen, Moch, Vogt

  b) Fully inclusive

  Developed (NLL): Bonciani, Catani, Mangano, Nason '98
  Applied: Cacciari et al, Moch Uwer, Czakon AM

The relation between the two pictures is still unclear !

σ_{TOT}: highlights

From: Cacciari et al '08

$$\sigma_{t\bar{t}}^{\rm NLO}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 875~^{+102(11.6\%)}_{-100(11.5\%)}~({\rm scales})~^{+30(3.4\%)}_{-29(3.3\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm LO}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 583~^{+165(28.2\%)}_{-120(20.7\%)}~({\rm scales})~^{+20(3.4\%)}_{-19(3.3\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm CTEQ6.5}) = 908~^{+82(9.0\%)}_{-85(9.3\%)}~({\rm scales})~^{+30(3.3\%)}_{-29(3.2\%)}~({\rm PDFs})~~{\rm pb}$$

Effect on central values:

- FO NLO / FO LO: 50%
- NLL / FO NLO: 4%
- New NLO effects / FO NLO: 1-1.5% Czakon, AM
- Beyond NLL effects / FO NLO: 0.8% Moch, Uwer

Important: No genuine NNLO term is known (could easily give 5%) !

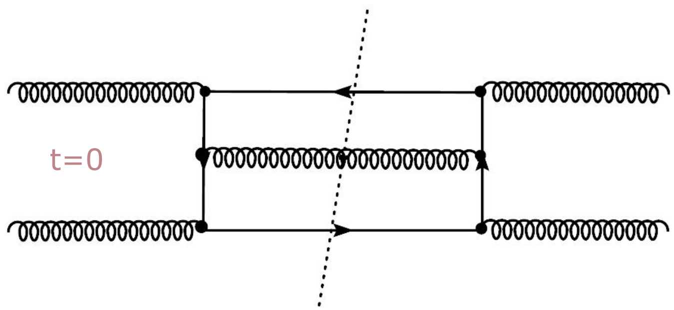
$$\sigma_{t\bar{t}}^{\rm NLO}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 927~^{+109(11.7\%)}_{-107(11.5\%)}~({\rm scales})~^{+11(1.2\%)}_{-12(1.3\%)}~({\rm PDFs})~~{\rm pb}$$

$$\sigma_{t\bar{t}}^{\rm LO}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 616~^{+172(27.9\%)}_{-126(20.5\%)}~({\rm scales})~^{+7.3(1.2\%)}_{-7.8(1.3\%)}~({\rm PDFs})~~{\rm pb}$$

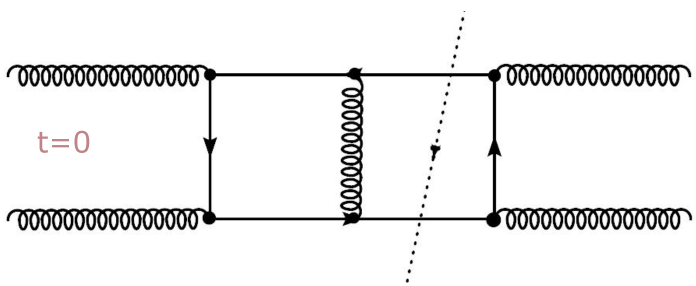
$$\sigma_{t\bar{t}}^{\rm NLO+NLL}({\rm LHC}, m_t = 171~{\rm GeV}, {\rm MRST2006nnlo}) = 961~^{+89(9.2\%)}_{-91(9.4\%)}~({\rm scales})~^{+11(1.1\%)}_{-12(1.2\%)}~({\rm PDFs})~~{\rm pb}$$

How complicated is the NLO?

Here are few sample diagrams at NLO:



+ crossed



+ crossed

- Note: these are 2 loop (cut) boxes with masses.
  Not studied before.

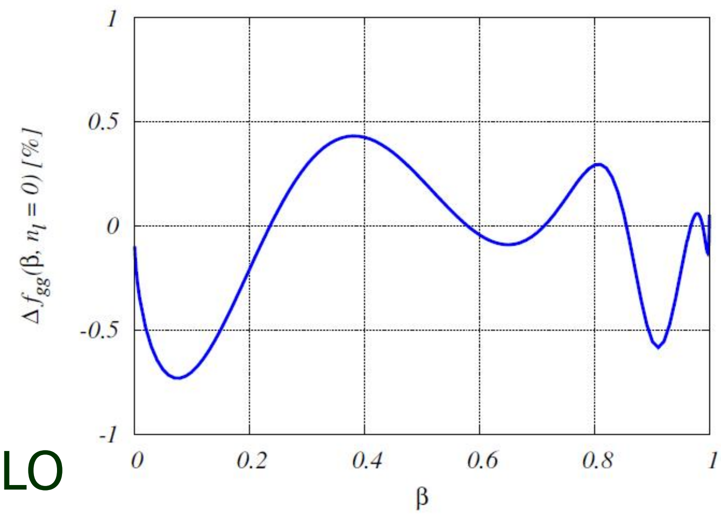
Main details of the new exact NLO calculation

- For 20 years σ_{TOT} was known as a numerically derived fit
- Newly calculated analytical results (new techniques):
  - The whole problem is mapped into 37 masters (real+virtual)
  - We find that the cross-section develops new unphysical singularities!
  - Appearance of elliptic functions,
  - We confirm the high numerical accuracy of the earlier FO results (< 1%)

NOTE: the qq-bar reaction
is too simple at NLO !

Only 4 massless masters appear ☺

gg represents the true complexity at NLO



Comparing our new analytic result with earlier numerical ones

Threshold expansion $\beta \to 0$: (i.e. $4m^2 \to s$)

$$
\begin{aligned}
f_{gg}^{(1)}(\beta) \;=\; & \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta)\left(\left(C_F - \frac{(N^2-4)\,C_A}{2(N^2-2)}\right)\frac{\pi^2}{2\beta} + 2C_A\log^2(8\beta^2) - \frac{(9N^2-20)C_A}{N^2-2}\log(8\beta^2)\right. \\
& + C_A\left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2\right) \\
& \left. + C_F\left(-5+\frac{\pi^2}{4}\right) + o(\beta)\right) . \qquad (27)
\end{aligned}
$$

Extraction of the constant in the threshold limit:

$$
C_A\left(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2\right) + C_F\left(-5+\frac{\pi^2}{4}\right)
$$

$$
= \frac{1111}{21} - \frac{283\pi^2}{168} + \frac{15\log 2}{7} - 6\log^2 2 \simeq 34.88\,,
$$

Czakon, AM '08

Nason, Dawson, Ellis '89

$$
\frac{768\pi}{7} a_0^{gg} \simeq 37.25\,.
$$

X-section better than 1%. But the constant in gg is 7% different.

Turns out, it is all consistent …

Hagiwara et al. '08

Significant (and unexpected) effect for threshold resummation!

More on resummation in top

From resummation, the following 2 loop logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\rm Born}(\beta) + \frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\rm Born}(\beta)\left(4608\log^4\beta + 1894.9\log^3\beta - 3.4811\log^2\beta + \mathcal{O}(\log\beta)\right)$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384 C_3$

where: $C_3 = 37.23$ As extracted from NDE '89 and used in ALL resummation literature

$C_3 = 34.88$ The exact value just recently derived Czakon, AM '08

Therefore the coefficient of $\ln^2(\beta)$ becomes -912.35

Note: the reason is pure numerics!

i.e. a change by a factor of 260 !

More on resummation in top (2)

The changes discussed so far are purely due to numerics.

However: there is another modification compared to earlier literature

Exponentiation in Mellin space: (1) $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho.$ $\rho = 4m^2/s$ (2)

(3) $\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N)$ (4) $\sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\ \sigma_{ij,\mathbf{I}}^{\mathrm{H}}\ \Delta_{ij,\mathbf{I}}(N)$

We were the first to point out σ^H depend on the color state of the heavy quark pair. We calculated the two coefficients.

Change in the gg Sudakov resummed X-section: compare to Bonciani et al '98

C_3 numerics: -5%,
color singlet channel: -12%,
color octet channel: -3%,

$$\begin{aligned}
\sigma_{gg}^{\mathrm{H\,(BCMN)}} &= 1 + \frac{\alpha_s}{\pi}\, 14.39 + \mathcal{O}(\alpha_s^2), \\
\sigma_{gg}^{\mathrm{H\,(BCMN)}}\big|_{C_3\ \mathrm{exact}} &= 1 + \frac{\alpha_s}{\pi}\, 12.04 + \mathcal{O}(\alpha_s^2), \\
\sigma_{gg,\mathbf{1}}^{\mathrm{H}} &= 1 + \frac{\alpha_s}{\pi}\, 9.16 + \mathcal{O}(\alpha_s^2), \\
\sigma_{gg,\mathbf{8}}^{\mathrm{H}} &= 1 + \frac{\alpha_s}{\pi}\, 13.19 + \mathcal{O}(\alpha_s^2),
\end{aligned}$$

Resummation - summary

These corrections are partially cancelled:

$$\sigma_{RESUM} = \sigma_{FO} + \sigma_{SUDAKOV} - \sigma_{OVERLAP}$$

That results in – (1-1.5)% shift.
Compare to 4% (from NLL) and 0.8% (from beyond NLL).

Implications to previous studies:

✓ Formally these effects are beyond NLL; yet significant numerically

✓ Incorrect beyond NLL (only one such study Moch, Uwer '08)

The big question is: why such sensitivity to the resummation?

And how relevant it is for the total cross section?

Work is in progress!

More numbers: newest PDF sets

Czakon, AM in progress

Comparison of central values for:

- m_{top}=172.4 GeV
- μ=m
- correct exact hard matching coefficients.

$\alpha_s(M_Z)$:

CTEQ 6.6: 0.118
MRST 2006 nnlo: 0.119
MSTW 2008 nnlo: 0.117
MSTW 2008 nlo: 0.120

MRST 2006 nnlo

NLO = 890 pb
NLO+NLL = 918 pb

CTEQ 6.6

NLO = 844 pb
NLO+NLL = 871 pb

MSTW 2008 nnlo

NLO = 857 pb
NLO+NLL = 885 pb

MSTW 2008 nlo

NLO = 906 pb
NLO+NLL = 935 pb

5.4 %

1.6 %

7.3 %

Conclusions

The summary from the new analytic calculation/updated resummation:

M. Czakon, A.M. (2008)

Conclusion #1: the earlier FO NLO calculations are of high quality 1%

Conclusion #2: the NLL resummation affected by our work (25-30% effect):

qq → tt unchanged at NLO/NLL (but likely at NNLO/NNLL)

Question: How to determine the scale uncertainty?

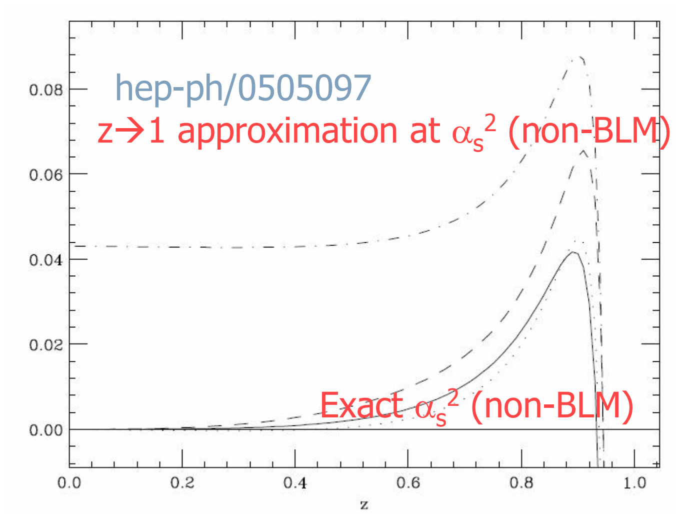
- The new set MSTW 2008 NNLO is (much) closer to CTEQ6.6 (for top-pair)

- New numbers will appear (in progress); trying to condense the field.

- Understanding true scale uncertainty requires full NNLO calculation !
- The appropriate observable is the total inclusive cross-section.
- Some NNLO terms can be obtained by truncating all-order resummation.
  - is this a systematic approximation?

In general, this is a poor approximation to fixed order calculations:

Photon spectrum in $B \rightarrow s+\gamma$:



Top X-section: NLO correction